An Analytical Method for the Assessment of Asset Price Changes in Capital Market

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Abstract

The success of any investment depends mainly on the value of asset which propels the entire financial strength of the organization. Therefore, the impacts on the value of asset prices of investors in capital market were analyzed; using four investment equations which were solved analytical by adopting the separable variable and Ito's lemma respectively. The computational and graphical results of stock variables and the effect of relevant parameters are well discussed.

Keywords: *Asset price, Stochastic system, Deterministic system and Stock returns*

1.1 Introduction

In financial markets generally an asset is one of key factor an investor would not want to play with due to its benefits in the life of every trader. Therefore valuation of asset is an act to assess market value of asset prices. It has become crucial factors in driving economic fluctuations, allocating economic resources across sectors with time and also influencing the financial strength of the entire system which yields levels of returns. Return on investment for capital market is a measure to properly evaluate gain of an investment. It is a major parameter often used by a trader or investors to regulate profitability of expenditure.

However, considering problem of this nature needs analytic approach, which can give exact solutions for proper mathematical predictions. It is essential to study asset valuation related problems, well formulated and accurate analytical solutions in order to measure realistic valuation results; therefore, the analytical solution is adopted based on the specific feature of the problem.

On the other hand, scholars has written widely on stock market prices such as [1,2, 3] etc. So in the work of [4] they considered the unstable nature of stock market forces using proposed differential equation model. While [5] studied stability analysis of stochastic model of price changes at the floor of a stock market. In their research précised conditions are obtained which determines the equilibrium price and growth rate of stock shares.

[6] Examined the stochastic analysis of the behavior of stock prices. Results showed that the proposed model is efficient for the prediction of stock prices. In the work of [7] they look at the stochastic model of some selected stocks in the Nigerian Stock Exchange (NSE), in their research the drift and volatility coefficients for the stochastic differential equations were determined.[8] built the geometric Brownian Motion and studied the accuracy of the model with detailed analysis of simulated data.

More so [3] worked on stochastic model of the fluctuation of stock market price is considered. Here conditions for determining the equilibrium price, sufficient conditions for dynamic stability and convergence to equilibrium of the growth rate of the value function of shares. On the other hand, [9] considered a stochastic model of price changes at the floor of stock market. In their research the equilibrium price and the market growth rate of shares were determined.

See [10] for considerable extensions and constrains subsequently in this particular area of study. [12] Explored the stochastic analyses of Markov chain in finite states. Their work replicated the use of 3-states transition probability matrix which enables them to proffer precise condition of obtaining expected mean rate of return of each stock.

In this paper, we considered two deterministic and stochastic time investment systems where we assumed stock return to follow multiplicative and multiplicative inverse trends respectively. These equations were solved and four different solutions obtained. So applying initial stock price and other parameter values into the solutions; the value of asset prices were predicted.

The aim of this paper is first; present an accurate analytical solution and predicting the value of assets prices of future investors as a time varying investments. However, to the best of our knowledge this is the first study to assume stock return to follow multiplicative and multiplicative inverse trends to measure the value of asset prices. To this end we extended the work of [6] by considering multiplicative and multiplicative inverse trends to assess the value of asset prices.

This paper is arranged as follows: Section 2.1 presents problem formulation, method of solution is seen in Subsection 2.1.1, Results are seen in Section 3.1, while the discussion of results is presented in Section 3.1.1 and paper is concluded in Section 4.1.

2.1 Problem Formulation

We consider a long and short trading period where investors will be faced with so many decisions on how to minimize loss and maximize profit. Assuming rate of return grows in two ways namely: multiplicative and multiplicative inversely trends at time t . This assertion is good because investors observes prices and takes action in discrete time periods $t = 0,1,2,3,4,...,T$,the factors underlying price changes are very uncertain and are described in probability terms,[13].Hence the rate of return is defined as follows:

$$R_t := \lambda_1\lambda_2,\ldots \text{where } t = 1,2\ldots \tag{1.1}$$

$$R_t := \left(\lambda_1\lambda_2\right)^{-1},\ldots, \text{where } t = 1,2.. \tag{1.2}$$

Thus the stochastic process describing the process is of the form:

$$dS(t) = \lambda dt + \beta dZ^{(1)}(\mathrm{t}) \tag{1.3}$$

where λ is an expected rate of returns on stock, β is the volatility of the stock , dt is the relative change in the price during the period of time and $Z^{(1)}$ is a Wiener process . Using (1.1)-(1.3) gives the following mathematical structure:

$$\frac{dS_1(t)}{dt} = \left(\lambda_1\lambda_2\right)S_1(t) \tag{1.4}$$

$$dS_2(t) = \left(\lambda_1\lambda_2\right)\mathrm{S}_2(t)dt + \beta S_2(t)dZ^{(1)}(\mathrm{t}) \tag{1.5}$$

$$\frac{dS_3(\mathrm{t})}{dt} = \left(\lambda_1\lambda_2\right)^{-1}S_3(t) \tag{1.6}$$

$$dS_4(\mathrm{t}) = \left(\lambda_1\lambda_2\right)^{-1}S_4(t)dt + \beta S_4(t)dZ^{(1)}(\mathrm{t}) \tag{1.7}$$

Where $S_1(\mathrm{t}), S_2(\mathrm{t}), S_3(\mathrm{t})$ *and* $S_4(\mathrm{t})$ are underlying stocks with the following initial conditions:

$$S_1(0) = S_0 \ , t > 0 \tag{1.8}$$

$$S_2(0) = S_0 \ , t > 0 \tag{1.9}$$

$$S_3(0) = S_0 \ , t > 0 \tag{1.10}$$

$$S_4(0) = S_0 \ , t > 0 \tag{1.11}$$

Though, the price evolution of a risky assets are usually modeled as the trajectory of a risky assets that are usually of a diffusion process defined on some underlying probability space ,with the geometric Brownian motion the paramount tool used as the established reference model, [13].

Theorem 1.1: (Ito's formula) Let $\left(\Omega,\beta,\mu,\mathrm{F}(\beta)\right.$ be a filtered probability space $X = \{X, t \geq 0\}$ be an adaptive stochastic process on $\left(\Omega,\beta,\alpha,\mathrm{F}(\beta)\right)$ processing a quadratic variation(X) with SDE defined as:

$$dX(t) = g\left(t, X(t)\right)dt + f\left(t, X(t)\right)dW(t)$$

$t \in \Re$ and for $u = u\left(t, X\left(t\right) \in C^{1\times 2}\left(\Pi \times \square\right)\right)$

$$du\left(t, X\left(t\right)\right) = \left\{\frac{\partial u}{\partial t} + g\frac{\partial u}{\partial x} + \frac{1}{2}f^2\frac{\partial^2 u}{\partial x^2}\right\}d\tau + f\frac{\partial u}{\partial x}dW\left(t\right)$$

2.1.1 Method of Solution

The model (1.4)-(1.7) consist of a system of variable coefficient differential equations and stochastic differential equations whose solutions are not trivial. We adopts the methods of variable separable and Ito's lemma in solving for $\mathrm{S}_1(t)$, $S_2(t)$, $S_3(t)$, *and* $S_4(t)$ To tackle this problem we note that

$\mathrm{S}_1(t)$, $S_2(t)$, $S_3(t)$ and $S_4(t) < \infty$ for all $r \in [0,1)$

From (1.4), this equation is solved using separable variable, hence

$$\frac{dS_1(t)}{S_1(t)} = \left(\lambda_1\lambda_2\right)dt$$

Integrating both sides gives

$$\int \frac{dS_1(t)}{S_1(t)}dS = \int (\lambda_1\lambda_2)dt + \phi \text{ , } \ln \mathrm{S}_1(t) = \lambda_1\lambda_2 t + \ln\phi$$

$$\ln\left(\frac{S_1(t)}{\phi}\right) = \lambda_1\lambda_2 t$$

Taking log of the both sides the following

$$S_1(t) = \phi e^{\lambda_1\lambda_2 t} \tag{1.12}$$

Applying the initial condition in(1.8) yields

$$S_1(t) = S_0 e^{\lambda_1\lambda_2 t} \tag{1.13}$$

From (1.5) , Thus, generalizing and considering a function $f\left(S_2(t), t\right)$,hence it has to do with partial derivatives. Expansion of $f\left(S_2(t), \mathrm{dS}_2(\mathrm{t}), t + dt\right)$ in a Taylor series about $\left(S_2(t), t\right)$ gives

$$df = \frac{\partial f}{\partial S_2(t)}dS_2(t) + \frac{\partial f}{\partial t}dt + \frac{1}{2}\frac{\partial^2 f}{\partial S_2{}^2(t)}dS_2^2(t) + \ldots \tag{1.14}$$

Substituting(1.5) in (1.14) gives

$$df = \frac{\partial f}{\partial S_2(t)}\left\{\beta S_2(t) dZ^{(1)}(\mathrm{t}) + \left(\lambda_1\lambda_2\right) S_2(t) dt \quad\right\} + \frac{1}{2}\frac{\partial^2 f}{\partial {S_2}^2(t)} dS_2^2(t)\,\mathrm{d}S_2^2(\mathrm{t})$$

$$df = \beta S_2(t)\frac{\partial f}{\partial S_2(t)} dZ^{(1)}(t) + \left(\left(\lambda_1\lambda_2\right) S_2(t)\frac{\partial f}{\partial S_2(t)} + \frac{1}{2}\beta^2 S_2^2(t)\frac{\partial^2 f}{\partial {S_2}^2(t)} dS_2^2(t)\right) dt$$

Now considering the SDE in(1.5)

Let $f\left(S_2(\mathrm{t})\right) = \ln S_2(t)$,the partial derivatives becomes

$$\frac{\partial f}{\partial S_2(t)} = \frac{1}{S_2(t)},\ \frac{\partial^2 f}{\partial S_2^2(t)} = -\frac{1}{S_2^2(t)},\ \frac{\partial f}{\partial t} = 0 \tag{1.15}$$

According to theorem 1.1(Ito's),substituting(1.14) and simplifying gives

$$df = \left\{\left(\lambda_1\lambda_2\right) - \frac{1}{2}\beta^2\right\} dt + \beta dZ^{(1)} \tag{1.16}$$

Since the RHS of (1.16) is independent of $f\left(S_2(t)\right)$,the stochastic is computed as follows:

$$f(S_2(t) = f_0 + \int_0^t\left\{\left(\lambda_1\lambda_2\right) - \frac{1}{2}\beta^2 \quad\right\} dt + \int_0^t \beta dZ^{(1)}(t)$$

$$= f_0 + \left\{\left(\lambda_1\lambda_2\right) - \frac{1}{2}\beta^2\right\} t + \beta dZ^{(1)}(t), \text{Since } f\left(S_2(t)\right) = \ln S_2(t) \text{ a found solution for } S_2(t) \text{ becomes}$$

$$\ln S_2(t) = \ln S_2 0 + \left\{\left(\lambda_1\lambda_2\right) - \frac{1}{2}\beta^2\right\} t + \beta dZ^{(1)}(t), \ln S_2(t) - \ln S_2 0 = \left\{\left(\lambda_1\lambda_2\right) - \frac{1}{2}\beta^2\right\} t + \beta dZ^{(1)}(t)$$

$$\ln\left(\frac{S_2(t)}{S_2 0}\right) = \left\{\left(\lambda_1\lambda_2\right) - \frac{1}{2}\beta^2\right\} t + \beta dZ^{(1)}(t)$$

$$S_2(t) = S_0 \exp^{\left\{\left(\left(\lambda_1\lambda_2\right) - \frac{1}{2}\beta^2\right) t + \beta dZ^{(1)}(\mathrm{t})\right\}} \tag{1.17}$$

This is the complete solution stock market prices when the return rate is multiplicative trend

From (1.6), this equation is solved using separable variable ,hence

$$\frac{dS_3(t)}{S_3(t)} = \left(\lambda_1\lambda_2\right)^{-1} dt$$

Integrating both sides gives

$$\int \frac{dS_3(t)}{S_3(t)} dP = \int \left(\lambda_1\lambda_2\right)^{-1} dt + K_1 \text{ , } \ln S_3(t) = \left(\lambda_1\lambda_2\right)^{-1} t + \ln K_1$$

$$\ln\left(\frac{S_3(t)}{K_1}\right) = \left(\lambda_1\lambda_2\right)^{-1} t$$

Taking ln of both sides gives

$$S_3(\mathrm{t}) = \mathrm{K}_1\, e^{(\lambda_1\lambda_2)^{-1} t} \tag{1.18}$$

Applying the initial condition in (1.10) yields

$$S_3(\mathrm{t}) = S_0 e^{(\lambda_1\lambda_2)^{-1} t} \tag{1.19}$$

where S_0 is the initial price of asset at time, t .

considering a function $f\left(S_4(t), t\right)$,hence it has to do with partial derivatives. Expansion of $f\left(S_4(t), \mathrm{dS}_4(\mathrm{t}), t+dt\right)$ in a Taylor series about $\left(S_3(t), t\right)$ gives

$$df = \frac{\partial f}{\partial S_4(\mathrm{t})} dS_4(\mathrm{t}) + \frac{\partial f}{\partial t} dt + \frac{1}{2} \frac{\partial^2 f}{\partial S_4{}^2(\mathrm{t})} dS_4^2(t) + \ldots \tag{1.20}$$

Putting (1.6) in (1.20) gives

$$df = \frac{\partial f}{\partial S_4(\mathrm{t})}\left(\beta S_4(t)\,\mathrm{dZ}^{(1)}(\mathrm{t}) + \left(\lambda_1\lambda_2\right)^{-1} S_4(\mathrm{t})\,\mathrm{dt}\right) + \frac{\partial f}{\partial t} dt + \frac{1}{2}\frac{\partial^2 f}{\partial S_4{}^2(\mathrm{t})} dS_4^2(t) \tag{1.21}$$

$$= \beta S_4(t) \frac{\partial f}{\partial S_4(t)} dZ^{(1)}(t) + \left(\left(\lambda_1\lambda_2\right)^{-1} S_4(t) \frac{\partial f}{\partial S_4(t)} + \frac{1}{2}\beta^2 S_4^2(t) \frac{\partial^2 f}{\partial S_4{}^2(t)} dS_4^2(t)\right) dt$$

Let $f\left(S_4(\mathrm{t})\right) = \ln S_4(t)$, the partial derivatives becomes

$$\frac{\partial f}{\partial S_4(\mathrm{t})} = \frac{1}{S_4(\mathrm{t})}, \ \frac{\partial^2 f}{\partial S_4^2(\mathrm{t})} = -\frac{1}{S^2{}_4(\mathrm{t})}, \ \frac{\partial f}{\partial t} = 0 \tag{1.22}$$

According to theorem 1.1(Ito's),substituting(1.21) and simplifying gives

$$= \left(\left(\lambda_1\lambda_2\right)^{-1} - \frac{1}{2}\beta^2\right) dt + \beta dZ^{(1)} \tag{1.23}$$

Since the RHS of (1.22) is independent of $f\left(S_4(t)\right)$,the stochastic is computed as follows:

$$\text{a}\, S_4(t) = S_0 \exp^{\left\{\left((\lambda_1\lambda_2)^{-1} - \frac{1}{2}\beta^2\right)t + \beta dZ^{(1)}(\text{t})\right\}} \tag{1.24}$$

Equations (1.19) and (1.24) is an investment of stock (3) and of stock(4) whose rate of return is inversely multiplicative respectively at time, t .

3. 1 Results

This Section presents the graphical results for the problems in (1.4)-(1.7) whose solutions are in (1.13)-(1.24), Hence the following parameter values were used in the simulation study:

$\lambda_1 = 2.00, \lambda \ = 4.00, S_1 0 = 6.48, \ S_2 0 = 6.48, \quad S_3 0 = 6.49, S_4 0 = 6.49,$
$\beta = 1.00, \ dZ = 1.00, \ t = 1$

Table 1:the effect of stock return on deterministic system that follows multiplicative trend series

t	$\lambda_1\lambda_2$	$S_1(t)$	$\lambda_1\lambda_2$	$S_1(t)$	$\lambda_1\lambda_2$	$S_1(t)$
1	0.5	10.70	1.5	29.09	2.00	47.95
2	0.5	21.40	1.5	58.17	2.00	95.91
3	0.5	32.10	1.5	87.26	2.00	143.86
4	0.5	42.80	1.5	116.34	2.00	191.82
5	0.5	53.50	1.5	145.43	2.00	239.77
6	0.5	64.20	1.5	174.52	2.00	287.73
7	0.5	74.90	1.5	203.60	2.00	335.68
8	0.5	85.60	1.5	232.69	2.00	383.64
9	0.5	96.30	1.5	261.78	2.00	431.59
10	0.5	107.00	1.5	290.86	2.00	479.55

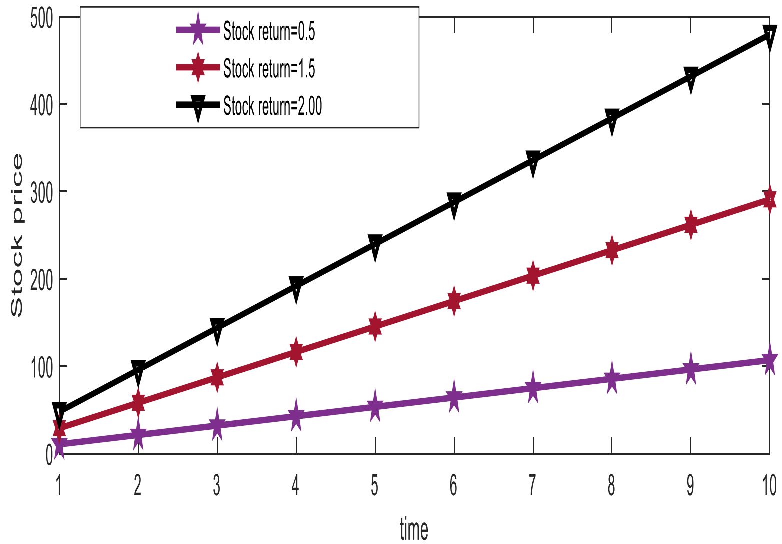

Figure 1: Graphical representation of stock return on deterministic system that follows multiplicative trend serie

Table 2:the effect of stock return on stochastic system that follows multiplicative trend series

t	$\lambda_1\lambda_2$	$S_2(t)$	$\lambda_1\lambda_2$	$S_2(t)$	$\lambda_1\lambda_2$	$S_2(t)$
1	0.5	13.95	1.5	32.33	2	51.20
2	0.5	10.70	1.5	29. 09	2	47.95
3	0.5	7.46	1.5	25.84	2	44.71
4	0.5	4.21	1.5	22.60	2	41.46
5	0.5	0.97	1.5	19.35	2	38.22
6	0.5	-5.28	1.5	16.11	2	34.87
7	0.5	-5.52	1.5	12.86	2	31.73
8	0.5	-8.77	1.5	9.62	2	28.48
9	0.5	-12.01	1.5	6.37	2	25.24
10	0.5	-15.26	1.5	3.13	2	21.99

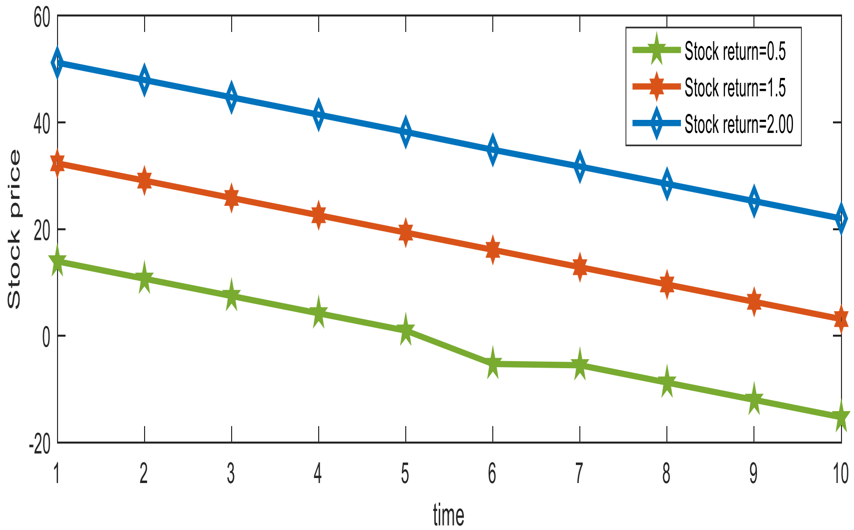

Figure 2: Graphical representation of stock return on stochastic system that follows multiplicative trend series

Table 3:the effect of stock return on deterministic system that follows multiplicative inverse trend series

t	$(\lambda_1\lambda_2)^{-1}$	$S_2(t)$	$(\lambda_1\lambda_2)^{-1}$	$S_2(t)$	$(\lambda_1\lambda_2)^{-1}$	$S_2(t)$
1	0.5	47.88	1.5	12.62	2	10.68
2	0.5	95.76	1.5	25.24	2	21.37
3	0.5	143.64	1.5	37.86	2	32.05
4	0.5	191.52	1.5	50.49	2	42.73
5	0.5	239.41	1.5	63.11	2	53.42
6	0.5	287.29	1.5	75.73	2	64.10
7	0.5	335.17	1.5	88.35	2	74.79
8	0.5	383.05	1.5	100.97	2	85.47
9	0.5	430.93	1.5	113.59	2	96.15
10	0.5	478.81	1.5	126.21	2	106.84

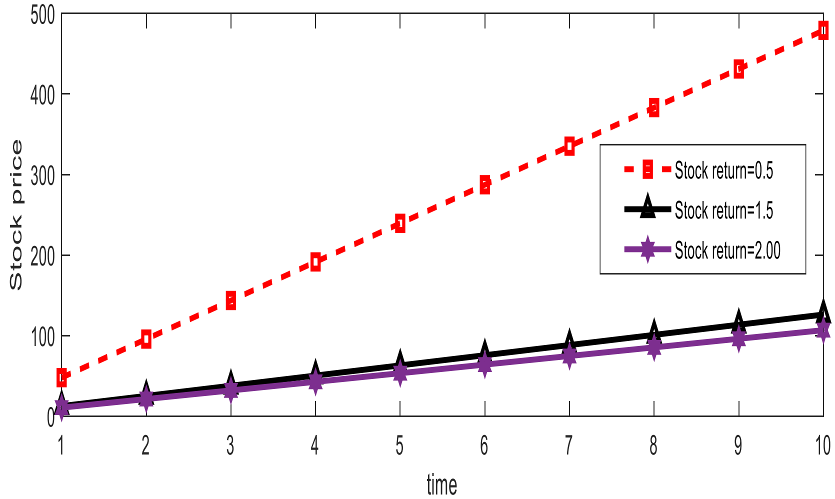

Figure 3: Graphical representation of stock return on deterministic system that follows multiplicative inverse trend series

Table 4:the effect of stock return on stochastic system that follows multiplicative inverse trend series

t	$(\lambda_1\lambda_2)^{-1}$	$S_2(t)$	$(\lambda_1\lambda_2)^{-1}$	$S_2(t)$	$(\lambda_1\lambda_2)^{-1}$	$S_2(t)$
1	0.5	51.12	1.5	15.86	2	13.92
2	0.5	47.88	1.5	12.62	2	10.68
3	0.5	44.64	1.5	9.38	2	7.44
4	0.5	41.40	1.5	6.14	2	4.20
5	0.5	38.16	1.5	2.90	2	0.96
6	0.5	34.92	1.5	-0.34	2	-2.28
7	0.5	31.68	1.5	-3.58	2	-5.52
8	0.5	28.44	1.5	-6.82	2	-8.76
9	0.5	25.20	1.5	-10.06	2	-11.10
10	0.5	15.48	1.5	-13.30	2	-15.24

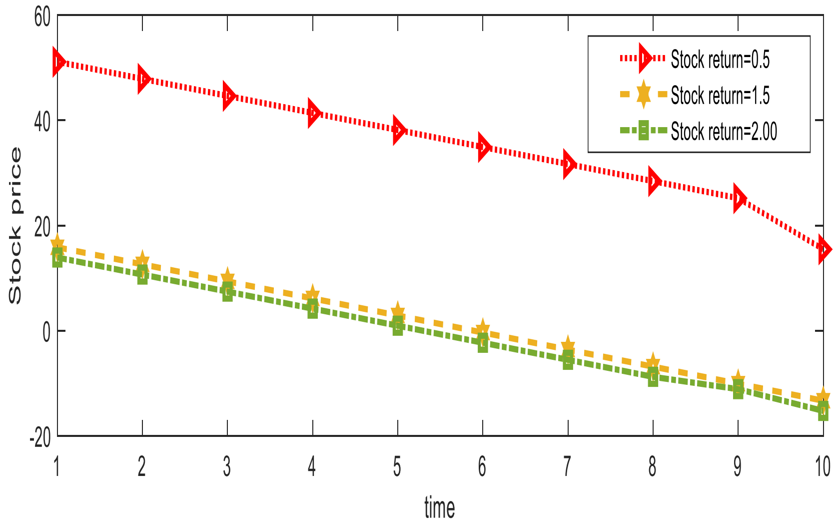

Figure 4: Graphical representation of stock return on stochastic system that follows multiplicative inverse trend series

3.1.1 Discussion of Results

Tables 1 and 3 shows increase in the time increases the value of asset prices. This quite realistic, because as time continues to move forwards the value of asset keeps on increasing. This remark will only be of great benefit to investors who deals more on lands, houses etc. The two assets mentioned do not depreciate in value. That is to say that, deterministic systems of both multiplicative and multiplicative inverse trends consider long-term investment changes. However, this informs an investor of long-term changes as a result of value of asset. The Figures 1 and 3 are in line with the explanation above.

It can be seen in Tables 2 and 4 shows a little increase in time decreases the value of asset for short-term business plans. The value of assets under this category is as follows: cars, furniture's, fridges and generator sets etc depreciates in value as time continue to grow. The negative signs seen in column 3 of Table 2 and columns 5 and 7 respectively of Table 4 are valuable assets depreciating at time, t. In all, stochastic systems of both multiplicative and multiplicative inverse trend series consider more of short-term investments. The explanations of Tables 2 and 4 holds for Figures 2 and 4 respectively.

4.1 conclusions

This paper investigated long and short term investment plans as it affects the value of asset prices. The proposed equations were accurately solved to obtain four different solutions. The solutions were varied with different stock return parameter and results obtained as follows: The analytical solution of problem shows:

i) A little increase in time increases the value of asset prices for deterministic system as it follows multiplicative and multiplicative inverse trends.

ii) An increase in time decreases the value of asset for stochastic systems when both follow multiplicative and multiplicative inverse trends.

iii) Deterministic systems consider long-term investment returns while stochastic systems consider short-term investment returns.

Therefore, combination of two different systems grantees better modeling results for the purpose of decision making. Finally, solving the deterministic and stochastic systems as coupled systems will be an interesting area of study.

Conflicting interest

We do not have any conflicting interest

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